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ABSTRACT

The projection of shadows task developed by Inhelder and Piaget was used to identify error patterns and solution strategies used by diverse populations in a sequence of proportion tasks. In Study A, two group tests were administered to 607 students from science and mathematics classes in grades 9 and 10. A sub-sample of 69 students was involved in clinical interviews on three Inhelder tasks. Classification of students was higher as level of course and grade increased. In Study B, a group test of direct proportions was administered to 1,200 students enrolled in college-bound mathematics or science classes in grades 10-12. A random sample of 128 successful students was interviewed on the projection of shadows task. Over 76% of these highly selected students were still unable to recognize inverse proportions in a novel task. Sex differences found on the proportions test were not found on the shadows task. In Study C, 30 gifted middle school students were given the various tests and tasks. The percentage at the Piagetian formal operational stage differed on various measures. Subtask data and interpretations are discussed for each study. (MNS)

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AN IN-DEPTH ANALYSIS OF THE PROJECTION OF SHADOWS TASK

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Our use of the projection of shadows task (Inhelder and Piaget, 1958) was designed to identify error patterns and solution strategies used by diverse populations in a sequence of proportion tasks. Many investigators (e.g., Karplus et al., 1977a; Goodstein & Boelke, 1980; Linn & Pulos, 1983; Hart, 1978) have been particularly interested in assessing proportional reasoning and have done so with paper-pencil measures. Linn and Pulos, like Siegler (1981), designed a task with forced choices--although their questions related to a version of a pictured balance beam while Siegler's was based on a demonstration of a version of the projection of shadows task. Hart who used Tall-Short (Karplus et al., 1975) and a "fish-feeding" problem from Piaget et al. (1968) identified possible error-patterns by interviewing secondary school children. Subsequently, those responses were used to design a paper-pencil test on ratio. In a study of Nigerian students, Fajemidagba (1983) assessed Piagetian stage using Longeot and the TOLT, and capability on ratio and proportion on a sequence of mathematics items. The items ranged from first order direct ($x/y = k$) to second order inverse ($xy^2 = k$) and were structured in either concrete or symbolic form. Although all students (N =200) had been instructed in content requiring direct and inverse proportion (i.e., $xy = k$), performance on all but first order direct proportion items was low across all groups. Fajemidagba's work is unique in its assessment of diverse levels of performance beyond direct proportion in the content area of mathematics. His findings on the difficulty of tasks involving inverse proportions are not unexpected.

Rogers (1977) assessed high school juniors' ability to respond successfully to four interview tasks, each of which required the use of proportions. The tasks could be solved by strategies based on either first order direct, second order direct, first order inverse or second order inverse proportions.

Rogers found that only 25 percent of his sample (N = 48) succeeded on first order inverse proportion and only 27 percent on the second order direct proportion task.

Yet ratio and proportion are key conceptual strands in both mathematics and science curricula in secondary school. Moreover, inverse proportions are repeatedly found in scientific formulas in the subject matter of chemistry and physics. Thus the analysis of strategies and error patterns with respect to this ability is of particular interest to both researchers and practitioners. In this context the projection of shadows task is particularly useful. Inhelder and Piaget (1958) used three tasks (the balance beam, the inclined plane and the projection of shadows) to assess proportional reasoning. All three tasks were thought to require the use of inverse proportions. However, subjects who succeeded on the balance beam and inclined plane task did not necessarily succeed on the projection of shadows task. The balance beam, in particular, has been reported to be easier than the shadows task. Piaget's analysis of the Genevan results (1958:199, 205) referred to two differences in the demands of the tasks. First, the shadows task has a spatial, geometric characteristic. Thus, he suggested a possible interaction between spatial ability and performance on the task. Second, the solution of the shadows task requires an understanding of proportion as a function while the balance beam task requires that the student establish an equivalence between two relations. In a later group of studies, Piaget et al. (1968) investigated the development of the function concept. In a section devoted to a study of inverse proportions on a balance beam task, the investigators found that successful students often found the constant product ($w \cdot d = k$) by taking the complement of the inverse. Yet it is exactly this understanding of the functional relationship inherent in inverse proportions which is so important in mathematics and science curricula. The projection of shadows task is

ideally suited for use in eliciting thinking patterns related to this functional relationship.

However, a search of the literature reveals that the projection of shadows is one of the less frequently used of the original Inhelder and Piaget tasks. Neimark (1975) reported only four uses of the shadows task. The obvious reason for avoiding it is the impossibility of repeatedly using the equipment, as depicted in Inhelder and Piaget. Lovell (1961) acknowledged the problem in his report of the use of ten tasks, including the shadows, from Inhelder and Piaget. Some researchers have modified the original task (apparatus and administration) in ways which require less (Siegler, 1981) or complex but different (Rogers, 1977) capabilities of subjects. Others (Schwebel, 1972; Bart & Airasian, 1974) have reported results on a replication of the Inhelder task. However, differences in reported task variables make comparison of results difficult. For example, Bart & Airasian used rings of one, two, three and four inches respectively. Hart's (1982) work in analyzing direct proportions indicates that use of small numbers or those easily referred to a one unit base changes the nature of the task to one which may be solved by additive processes. Finally, some researchers (Pallrand, 1979; Piburn, 1980) have reported the use of the shadows as depicted in Inhelder and Piaget but provide no details on the apparatus, the sequence of prompts or the specific scale used for classification. Given the inherent design problems of the Inhelder and Piaget equipment and the incomplete nature of the sample scripts, it is difficult to accept research results which neither allude to apparatus problems nor note the need for careful planning of interview details to obtain the sequence of abilities referred to by Inhelder and Piaget. Unfortunately, errors of varying significance are found in the literature on proportion. In Voyat's (1982) text on Piaget's work, the final sentence in the description of the

projection of shadows task contains a serious error. The 3B subject's behavior is described and the general law in question is stated as "The size of the shadow is directly (our italics, proportional to the distance between rings and light source." (p. 188). There is no hint to the reader that inverse proportion is involved. Similarly on page 187, the laws for the equilibrium in the balance task is misstated as $\frac{W}{w'} = \frac{L}{L'}$, a direct proportion.

The background presented thus far indicates the potential inherent in the use of the shadows task as a researcher's diagnostic tool. It also points out some ambiguities and fallacies in reported uses of the task. Hence, the need for further attention to this measure and a study of the implications of data collected from it is apparent.

The Shadows Task

The Apparatus. The shadows task, whose differences from other proportionality tasks were reported by Inhelder and Piaget (1958; p. 205) and reaffirmed by others (Lovell, 1961), includes several sub-tasks, each of which demands a different level of understanding of proportion. Thus, it appears to offer an opportunity for diagnosis, as well as for stage-classification.

Our version of the shadows apparatus was designed to be a close replica of that depicted in The Growth of Logical Thinking from Childhood to Adolescence (1958). Two modifications were devised to ensure functionality: (1) concentric rings of different sizes suspended from various locations on a meter stick; and (2) an electric "point source" of light at a constant optical axis.

Figure 1 about here

The meter stick was labeled with two scales in increments of 2 cm--one with the zero point at the light and the other with the zero point at the screen. This was done to avoid cuing either screen or light as the appropriate reference point. A six-volt flashlight bulb connected to a six-volt dry cell was employed to ensure the formation of sharply outlined shadows. Solder (in wire form) was used to construct concentric rings of 3 cm, 6 cm, 9 cm and 12 cm diameter with stems of varying lengths calculated to cause the center of each to coincide with the optical axis when suspended from the nails on the meter stick. A black velvet cloth attached to the rear of the apparatus shielded the screen from the undesirable stray light present in most of the test environments encountered.

The Interview. Faithful adherence to both the Inhelder and Piaget protocols and Piagetian theory was attempted but ambiguities and lack of detail posed serious problems to reliability across interviewers. Thus, a detailed interview script was developed.

Figure 2 about here

The outline was designed to provide brief, but important, cues so that the interviewer could (1) quickly check typical student responses and (2) consistently choose tested, workable locations. For example, the locations in step 4 were selected because they gave clear images of shadows and those shadows did not exceed screen size. Notice that locations less than 42 cm from the light were not to be used since the resulting shadow of the 9 cm ring would rapidly exceed screen size. Different pairs of locations were listed in step 5 to allow for working instances of new trials. The same is true of step 7 although the alternate location was also used to counteract potential cross-talk among subjects drawn from the same class and school. The choices on the script were to be circled, checked or, in the case of steps 6 and 7,

locations written in, while the interview was occurring. Additional notes were to be written by the interviewer immediately after the subject left.

After the subject and the interviewer had talked for a few minutes in an "easing-in" setting, the subject was told that the interviewers were studying problem-solving strategies used by students, that their particular interest was in helping mathematics and science teachers understand different ways students approach problems. The functioning of the equipment was demonstrated to the subject with one ring in place. Then the subject was told that he or she would be asked to make predictions about shadows while the light was turned off and give reasons for the predictions. Then the light would be turned on so the subjects could check on the correctness of their predictions. In sub-task one, a 3 cm and a 9 cm ring were placed on a common nail and the subject was asked to predict the relative size of the shadows. "Will the shadows be equal or will one be larger than the other? If unequal, which ring will cast the larger shadow? Why?" The subjects who correctly related the size of the shadows to the size of the rings were classified as, at least, 2A. (It soon became clear that older subjects considered this task a superficial one and resisted the verbalization of a reason. Therefore, interviewers did not press subjects for a reason in this one instance.)

In sub-task two, the interviewer showed two 9 cm rings and made sure that subjects saw that these were the same. "Now I'll place these two rings on two different nails. Let's see, here and here will do." The interviewer used locations noted on the script. The questions asked previously were repeated. However, in this sub-task, subjects were required to give a reason. The light was turned on and prediction checked. Subjects who predicted shadow size of ring correctly and explained <u>either</u> that the shadow is larger for the ring further from the screen <u>or</u> that the shadow is larger for the ring closer to the light were classified as, at least, 2B. In

accord with the original Inhelder protocols, subjects were allowed a second trial on this sub-task if they predicted incorrectly on their first attempt. On the second trial, the interviewer (with the light out) moved the rings to two new locations. The potential for learning from the previous trial is an interesting aspect of the shadows task. Unsuccessful subjects sometimes not only corrected the prediction but also focused on the inverse proportion (i.e., shadow size increases as distance of the ring from the light decreases) which underlies this task. It is also true that the second trial on this sub-task allows the subject who had guessed that the wrong rings produce unequal shadows to now simply reverse that decision. In these cases, the next sub-task provides information on whether the subject had, indeed, learned from the feedback. Classification of subjects requiring a second trial was delayed until they responded to sub-task three.

In sub-task three, subjects were asked to place a 3 cm and a 9 cm ring on the hanger board so that the shadows would coincide or "cover each other exactly." An explanation was required before the light was turned on and a second trial was permitted for unsuccessful subjects or in cases where the interviewer needed to check on an incomplete verbalization by the subject. In such cases, the interviewer might ask "Are these the only locations which give these results?' Subjects who placed the rings in the right order but wrong locations consistently were classified as 2B. Even those who had required a second trial on sub-task two were assessed as using a distance/shadow size ratio concept although they were either (1) concentrating on finding a direct proportion or (2) completely unable to find a metric relationship.

Finally, subjects successful on the first three tasks were given the fourth sub-task. The interviewer placed a 6 cm ring at 36 (or 32) and subjects were asked to place a 9 cm and a 12 cm ring on other nails so that

all three shadows would coincide when the light was turned on. They were also asked to explain why these choices were made. A second trial was given to unsuccessful students and, in most cases, to check the successful subject's ability to generalize beyond one case. Success on this sub-task was evidence of 3B reasoning.

The term "EXIT" in Figure 2 should be interpreted to mean that a classification decision could generally be made at that point. However, to save embarrassment of subjects, interviewers proceeded with the next sub-task with students who were unable to perform the first two sub-tasks.

Interviewer Training. The team of clinical interviewers consisted of the researchers and advanced graduate students in mathematics or science education who were enrolled in a Piagetian research seminar. All viewed a researcher-produced videotape of a variety of concrete and formal operational subjects being administered the projection of shadows task. After the ensuing discussion and analysis of the videotape, each interviewer studied relevant transcripts in the original Inhelder and Piaget reference and prepared to role-play one of the described subjects. During the role-playing sessions, research team members alternated between serving as subjects and as interviewers. Finally, junior and senior high students (from schools not to be included in the research) were interviewed by team members. Videotapes of these interviews were analyzed to assist team members in details of the interview procedures. The data from these same tapes were also used to obtain a measure of inter-rater agreement on classification. An inter-rater agreement of .95 was obtained. Approximately 10 to 12 hours of training were required to train each interviewer to criterion performance.

Procedures. In studies A and B, all interviews were conducted either on the same day, or on two successive days, in a given school in a private room set aside for this purpose. In study C, interviews were conducted over

a three-day period, at the University where students were involved in a summer institute. Parental permission slips were used to inform both students and parents that (1) we were interested in learning more about the diverse ways young people think through science and mathematics-related problems and (2) no individual's results would be shared with school personnel or be made a part of any school record. This was reinforced verbally as soon as subjects entered the interview room. One team member served as a "checker." Both kept written records on the prepared script sheets. An audiotape recording was made of each interview and any written work produced by subjects was attached to the completed script sheet. Interscorer agreement was reached on classification immediately after the subject left the room. Any unusual cases were earmarked for later follow-up checking of tapes and script. In addition, the interviewer immediately taped verbal comments on the nature of that interview. Less than 6 percent of the cases were of this nature and all were from study B. All scripts were subsequently reviewed and checked by the researchers before data were coded.

Study A

The major purpose of the study (Farmer et al., 1982) was to obtain a measure of the construct validity of two paper and pencil group tests of concrete and formal operations. The two group tests, Sheehan's (1970) version of Longeot's three sub-tests and a set of three puzzles originally separately developed by Karplus, Lawson and Renner (Karplus et al., 1977b) were administered to 607 students from ninth and tenth grade science and mathematics classes. Approximately two-thirds of the class sections from which the students were drawn were courses for the college-bound student. A sub-sample of 69 students, drawn from the Formal-Formal, Formal-Concrete, Concrete-Formal and Concrete-Concrete classifications on the group tests, were involved in clinical interviews on three Inhelder (1958) tasks--the projection

of shadows, the solutions and the bending rods.

The findings of interest for this paper are results on the projection of shadows task and the contrast between results on that task and the solutions and bending rods (Table 1). These results agree with those of

Table 1 about here

researchers such as Lovell (1961) who found the projection of shadows task to be one of the more difficult Inhelder and Piaget tasks. However, since our sub-sample was intended to include equal numbers of subjects in the four cells (FF, FC, CF, CC), approximately 50 percent had also been assessed as concrete on the KLR battery which included Tall-Short, a measure of first order direct proportion. Since first order direct proportion is a logical prerequisite for inverse proportional reasoning, we conjectured that subjects screened on a measure such as Tall-Short should demonstrate proficiency at the 2B and 3A levels on the projection of shadows task. Furthermore, this study had included subjects in the first two years of high school, at least one-third of whom were in a lower-track mathematics science course (e.g., general mathematics or general science). The results on the projection of shadows task by grade and level of course (Table 2) showed a trend toward higher classification as level of course and

Table 2 about here

grade increase. Grade was found to correlate significantly with the group Piagetian measures. We conjectured that subjects who were enrolled in upper level courses in mathematics or science should not only be more successful on Tall-Short, but when screened on Tall-Short, should be

differentially proficient on the projection of shadows as a function of level of mathematics or science course presently studied. Our assumption was based on knowledge of the experience with proportions of each kind which is incorporated in sequential mathematics and science courses. These were the major questions to be investigated in the second study.

Study B

The major purpose of this study was to investigate the acquisition of first order inverse proportions by a college-bound sample of secondary students who had been successful on a measure of direct proportion. A secondary purpose was to investigate the relationships between the subject's level of acquisition of the proportionality schema and past experience in the learning of mathematics/science concepts.

A group test of direct proportions, Tall-Short, was administered to 1,200 students, all of whom were enrolled in a tenth, eleventh or twelfth grade college-bound mathematics and/or science class. The sample was reduced to 901, as a result of testing irregularities in one school. Of this group, 474 students (52.6%) were classified as formal operational, or equivalently, succeeded on the measure of direct proportions.

A random sample of these successful students (N = 128) was interviewed on the projection of shadows task. This task resulted in classifications of 24.3 percent formal (22-3B and 9-3A) and 75.7 percent concrete (92-2B and 5-2A). Of the relationships investigated, only number of prior mathematics courses correlated positively and significantly ($.186, \alpha < .05$) with Piagetian classification. In contrast, correlations of age ($.103, \alpha < .01$), grade ($.213, \alpha < .01$), sex ($-.167, \alpha < .01$), number of prior mathematics ($.349, \alpha < .01$) and science ($.258, \alpha < .01$) courses with classifications on Tall-Short were significant and positive (sex in favor of males).

The results on the projection of shadows task are most surprising and have major implications for both curriculum and instruction in mathematics and science, as well as posing some curious conjectures for researchers. The fact that over 76 percent of these highly selected students were still unable to recognize inverse proportions in a novel task suggests that they must have resorted to rote learning of comparable classroom tasks. Also, the shadow task is even more spatially-oriented than Tall-Short. Why are the sex differences present in the scores on Tall-Short not evident in the shadows data?

In a recent study which focused on proportional reasoning during adolescence and sex differences, Linn and Pulos (1983) found that males performed slightly better than females on the proportional reasoning measures but that these sex differences were not accounted for by various measures of aptitude, including spatial ability. In fact, Linn and Pulos found a lack of relationship among tasks that individually yielded sex differences. However, as in the studies reported in this paper, the researchers drew attention to the generally low attainment of proportional reasoning by both sexes. In our study, it seems that whatever factor(s) account for sex differences on Tall-Short, the effect of these in more complex proportion tasks is neutralized by success in the first order direct proportions task offered by Tall-Short.

In a post-hoc analysis of the interview data, the kind of responses given and the effect of success on a second trial were investigated. As can be seen in Figure 3, 73 percent (91) of those succeeding on sub-task two did so on one trial. This same group of subjects eventually yielded 19 of the 22-3B subjects.

Figure 3 about here

Recall that sub-task two merely requires that the student correctly predict whether two equal (diameter) rings at different locations produce equal or unequal shadows; and if unequal, correctly identify the ring which produces the larger (smaller) shadow. In fact, 12 subjects out of the 128 said the shadows would be equal. Ten of these subjects succeeded, as a result of feedback, on trial two and focused on the distance from the light. One of these 12 insisted that the shadows would be equal and one matched the wrong ring to the larger shadow.

We will return to reasons used by subjects in a later section. However, at this point, our analysis of the data focuses on the role of the number of trials needed for success. Notice that only 7 of the 22-3B subjects succeeded on the first trial for each sub-task. Half of the 3B subjects required two trials on each of sub-task three and sub-task four. Since subjects who had two trials got visual feedback on errors and successes, one might conjecture that these additional contrasting prompts are critical for eventual classification as 3B for many subjects. However, only 14 of 108 subjects (13%) succeeded on the second trial of sub-task three. For these subjects, the additional feedback was of little help. They either had no idea how to construct a metrical relationship or struggled with inappropriate rules. Such students knew that the 3 cm ring should be between the light and the 9 cm ring in order to produce coincident shadows but they were unable to predict locations based on a metric. A discussion of some of the stumbling blocks faced by high and low 2B subjects may be found in Farmer et al. (1981).

The literature on the projection of shadows task frequently alludes to the function of reasons associated with various sub-tasks. The protocols in Inhelder and Piaget (1958) refer to the subject's focus on the light

versus the screen in sub-task two and multiplicative versus additive reasons in sub-task three. Students who focus on distance from the screen have been characterized as those searching for a direct proportion--further from screen, larger shadow. Since visual feedback on this sub-task does not contradict this focus, such subjects, it is assumed, will try to find a metrical relationship for sub-task three based on the same concept. The tree diagram in Figure 4 illustrates a pattern of reasons for a sub-sample of subjects. Since 70 percent of the subjects were successful on trial #1 of sub-task two, data on this group of students is displayed in Figure 4.

Figure 4 about here

Over five-eighths of these subjects focused on the light in the successful completion of sub-task two. This is probably a higher percentage than would appear in the original secondary school population tested on Tall-Short. Notice that the additive reason is an unlikely one for both groups, again a probable result of using the screened sample. The reason designated as "other" included diverse mixed rules, guessing and "eyeballing" the distances. For example, a subject might attempt to recall the size of the shadow displayed on the feedback to sub-task two and "cut-and-try" a likely location. The multiplicative reason was used by half of the "light" subjects and half of the "screen" subjects on trial #1 of sub-task three. However, on trial #2, although both groups shifted from the multiplicative reason, the "screen" subjects were more likely to choose another reason. An example of the use of a multiplicative reason by screen respondents is the location of the 3 cm ring, which they stated had to produce the larger shadow, three times further from the screen than the 9 cm ring. Unfortunately, in such cases, the feedback following this prediction undermined the

confidence of these subjects. Thus, rather than seek other quantities to relate in a multiplicative way, they often abandoned multiplication and either tried a different metrical relationship or "eyeballed" the distance. Such subjects gave no indication of having considered the possibility of an inverse proportion. The data in Figure 4 appear to highlight the importance of the reasoning displayed on sub-task two to later, more complex tasks.

Study C

The major purpose of this study (Farmer, 1983) was to assess the cognitive development of a group of 30 gifted middle school students attending a summer institute on problem solving via microcomputers. This population consisted of 1 fifth grader, 3 sixth graders, 14 seventh graders and 8 eighth graders and included six females. Assessment measures included the ninth-twelfth grade version of the Henmon-Nelson Test (a deviation I.Q. test), Tall-Short, TOLT (Tobin & Capie, 1980), Longeot and the projection of shadows task.

All subjects were classified as formal operational on the Longeot while 87 percent (26) and 80 percent (24) were classified as formal by Tall-Short and the TOLT respectively. In contrast, the projection of shadows task resulted in classifications of 27 percent formal (one 3A and seven 3B) and 73 percent concrete (22 at stage 2B). Scatter plots of each possible pairing of the variables of age, I.Q., Longeot, Tall-Short, TOLT and projection of shadows obviated the need for any statistical search for correlations. (Figure 5 depicts the scatter plots of the projection of shadows data with data from each of the above variables.)

Figure 5 about here

Since 87 percent of these subjects had succeeded on Tall-Short, there is a fair approximation of the screen used in Study B. As in that study, we analyzed data on number of trials and a pattern of reasons used by subjects (Figure 6). A higher percentage of subjects (83% compared to 73%

Figure 6 about here

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in Study B) succeeded on sub-task two in one trial. However, surprisingly, none of these gifted middle school subjects succeeded on sub-task three on trial #1. On trial #2 of this sub-task, eight subjects (27% compared to 25% across both trials in Study B) were successful and, of that group, all but one attained 3B classification by succeeding on sub-task four. It is clear that these younger students with less exposure to mathematics and science instruction and, in most cases, with no instruction in mathematics and science beyond typical ninth grade courses, performed in a manner comparable to the older, more course-experienced subjects of Study B. As has been noted by Nagy and Griffiths (1982), developmental level and conceptual content appear to be associated in a clear but empirically weak manner. Yet the fact that none of these subjects succeeded on trial #1 of sub-task three may be related to lack of practice in courses where proportion is a major strand. If that is so, these subjects demonstrated great facility in constructing a metric based on feedback from trial #1.

Recall that four of these subjects did not succeed on Tall-Short. On Tall-Short, two of these subjects were classified as 2A (additive response) and the remaining two as 2B (initial use of a ratio but inability to use the two ratios in a proportion). All four of these subjects were classified as 2B on the projection of shadows task.

A comparison of the light versus screen responses given by 25 of these

subjects is displayed in Figure 7. A higher percentage of the Study C trial

Figure 7 about here

#1 subjects (98% compared to 73%) used the light as the focus on sub-task two. As before, more of the "light" respondents gave a multiplicative response in trial #1 of sub-task three. Again the additive reason, although selected by both "light" and "screen" subjects, is not a popular reason. The trend noted among Study B subjects occurs in a modest way in this study. On trial #2 of sub-task three, the "screen" subjects were more likely to shift away from a multiplicative reason. Furthermore, eight of the nine subjects successful on trial #2 were "light" subjects. Finally of the six subjects from this group who were eventually successful on sub-task #4 (Figure 7), five were "light" respondents.

Summary

When the results of all three studies are compared (Table 3), one

Table 3 about here

cannot help but notice that approximately three-fourths of the subjects in each study were able to demonstrate, at best, the application of the concept of direct proportion to a novel situation. Secondly, the expected distribution of subjects across the 2B and 3A levels did not occur. Instead, there was, in each study, a sharp decline in the percentage of successful subjects as the added criterion found in sub-task three had to be met. However, most of the students who succeeded on sub-task three were also able to perform successfully on sub-task four. The projection of shadows task differs from other Inhelder tasks in this

respect. In the solutions task for example, the criterion for 3B seems to constitute a major hurdle for most subjects. In the projection of shadows task, on the other hand, subjects who can identify the relationship between two rings and their shadows typically appear to be able to succeed on the three-ring task.

There are questions, even here, about the generalizability of the fourth sub-task. Because the rings used were 6 cm, 9 cm and 12 cm, the subject could locate the 12 cm ring by doubling and then find the mid-point. As noted earlier, Hart (1978) found that subjects who could apply proportions involving doubling and halving, could not necessarily apply proportions requiring other relationships. Yet even this relatively easy fourth sub-task, and the as-easy third sub-task, represented insurmountable tasks in each study.

Why do subjects who attend to a distance relation remain fixed on the same pair of distances? In this case the screen distance plus the light distance (to the object) are equal to the constant 100 cm. An analogous situation may shed some light on the problem. A student may be told that the sum of two numbers is 14. "If one of the numbers is represented by x, how can we represent the other in terms of x?" The question, which requires the student to represent one part in terms of the whole and the other part, is always a difficult one for algebra students. In this case this problem is compounded by the need to analyze a functional relationship and to coordinate the two features of sub-tasks one and two in a third problem.

Yet the ability tested in the projection of shadows is important in most of secondary mathematics/science curricula. Would prompts, other than the typical feedback, aid understanding? One possible strategy would be the use of a diagram with "light rays" sketched in

from the source to one of the rings. If this prompt were shown to the subject unsuccessful on trial #1 of sub-task three, would it trigger the concept of similar triangles? This is but one of the potential prompts which might be used to further diagnose first order inverse proportional reasoning. Instructional intervention, similar to that proposed by Hart (1978) for younger children, needs to be devised for upper level mathematics and science students. Curricular modification of the kind developed by Shayer and colleagues (Shayer & Adey, 1981) is in order for both mathematics and science.

TABLE 1

Classification by Three Inhelder and Piaget Tasks

Task	2A #	2A %	2B #	2B %	3A #	3A %	3B #	3B %	Total N
Solution	5	(7.2)	48	(69.6)	16	(23.2)	0	(0.0)	69
Rods	20	(30.0)	24	(34.8)	22	(31.9)	3	(4.3)	69
Shadows	7	(10.1)	59	(85.5)	1	(1.4)	2	(2.9)	69

TABLE 2

Shadows Results by Grade

Grade	2A #	2A %	2B #	2B %	3A #	3A %	3B #	3B %	Total N
9N	5	(25.0)	15	(75.0)	0	(0.0)	0	(0.0)	20
9R	2	(8.3)	20	(83.3)	1	(4.2)	1	(4.2)	24
10R	0	(0.0)	24	(96.0)	0	(0.0)	1	(4.0)	25
TOTALS	7	(10.1)	59	(85.5)	1	(1.4)	2	(2.9)	69

N = Non-Regents
R = Regents (college-bound)

TABLE 3

Projection of Shadows Results Across Three Studies

Study	2A #	2A %	2B #	2B %	3A #	3A %	3B #	3B %	Total N
A	7	10.1	59	85.5	1	1.4	2	2.9	69
B	5	3.9	92	71.8	9	7.1	22	17.2	128
C	0	0.0	22	73.3	1	3.3	7	23.3	30
Combined	12	5.3	173	76.2	11	4.8	31	13.7	227

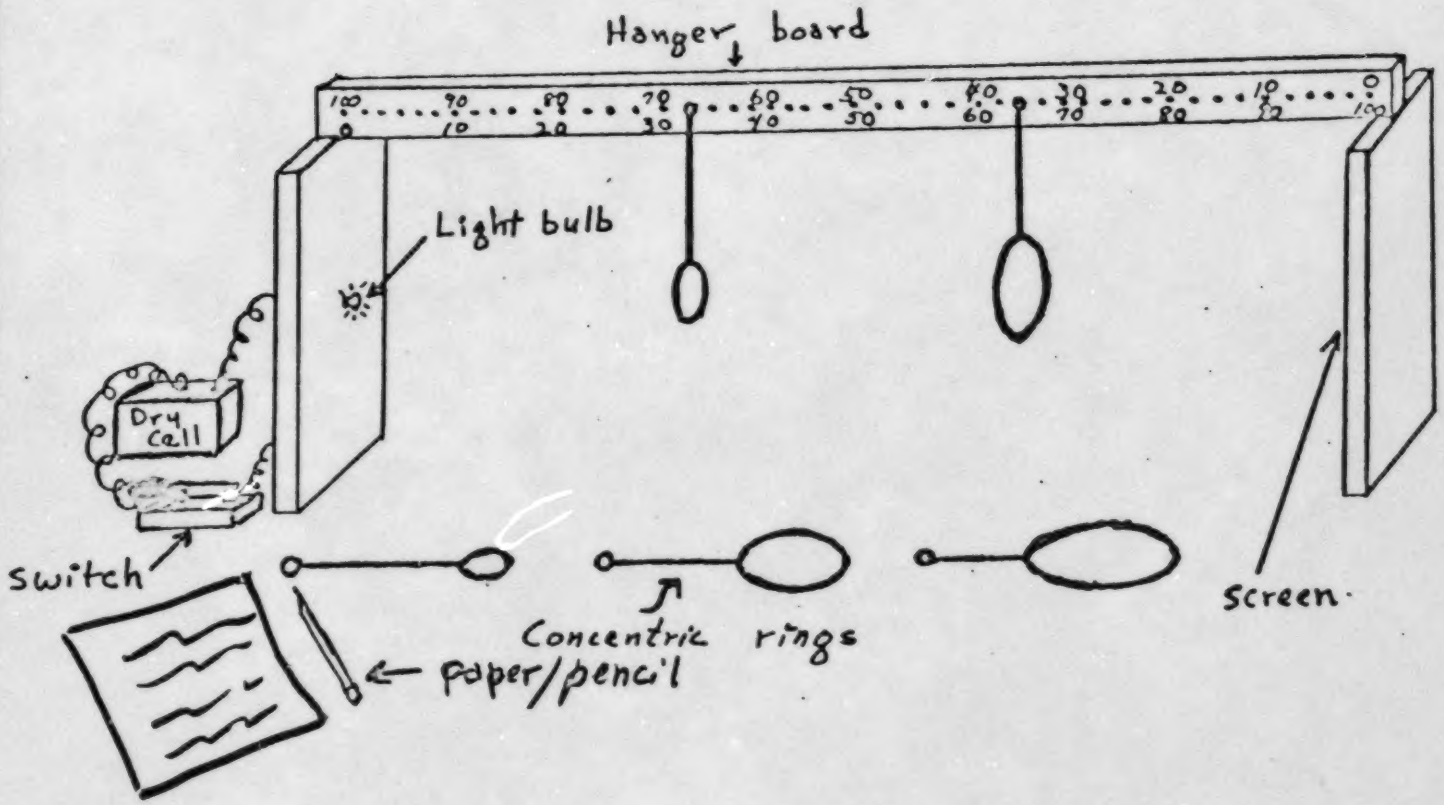

FIGURE 1: Projection of Shadows Apparatus

Interviewer_____ Student_____ Code #_____
Date_____ Time_____
Reliability Checker_____ Classification_____

Script and Prompts

1. Introduction
2. Show Rings; State size of diam.; Show shadow; Indicate numbers on scale (both directions); Tell students paper/pencil available.
3. State procedure (Predict, explain with light off, check with light on)

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4. 3 cm ring and 9 cm ring - at same distance (56 from light: anywhere from 42→70)
   (A) Larger, smaller, equal shadows? Which ring, which shadow?
   (B) Why do you think so?

   Check with light. Turn light off.

CHECKLIST

	Trial #1 R	Trial #1 W	Trial #2 R	Trial #2 W
A.				
B. Guess				
shadow ∝ diam.				
other				

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5. Two 9 cm rings at different distances--
   (48, 76) or (46, 78) or (42, 82)
   (A) Larger, smaller, equal shadows? Which ring, which shadow?
   (B) Why?

   Check with light. If wrong, turn light off, ask if S can explain, then try a second time with new distance. Ask A & B.

   Check with light. If wrong, EXIT.

	Trial #1 R	Trial #1 W	Trial #2 R	Trial #2 W
A.				
B. Guess				
Dist. from light				
Dist. from screen				
other				

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6. (A) S (or I as told by S) to place 3 cm and 9 cm to get shadows which "cover each other"
   (B) Why did you choose those places?

   Check with light. If wrong, turn off light and perform A and B again. (Try again to confirm judgment.)

   Check with light. If wrong, EXIT.

*(Record # from light.)

	Trial #1	Trial #2
A. 3 cm ___*		*
9 cm ___*		*
B. Additive		
Mult.		
"Cut-and-try"		
Meas. from screen		
Other		

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7. One 6 cm ring at 36 (or 32)
   (A) Ask S to place 9 cm and 12 cm to get shadows which "cover each other"
   (B) Why did you choose those places?

   Check with light. If wrong, turn off light and try A and B again. (Try again to confirm judgments.)

   Check with light. If wrong, EXIT.

*(Record # from light.)

	Trial #1	Trial #2
A. 9 cm ___*		*
12 cm ___*		*
B. doubling, then halved		
other ratio/ proportion		
other		

Additional anecdotal notes on reverse side? (Please circle.) Yes No

FIGURE 2: Test Protocol for Projection of Shadows Task

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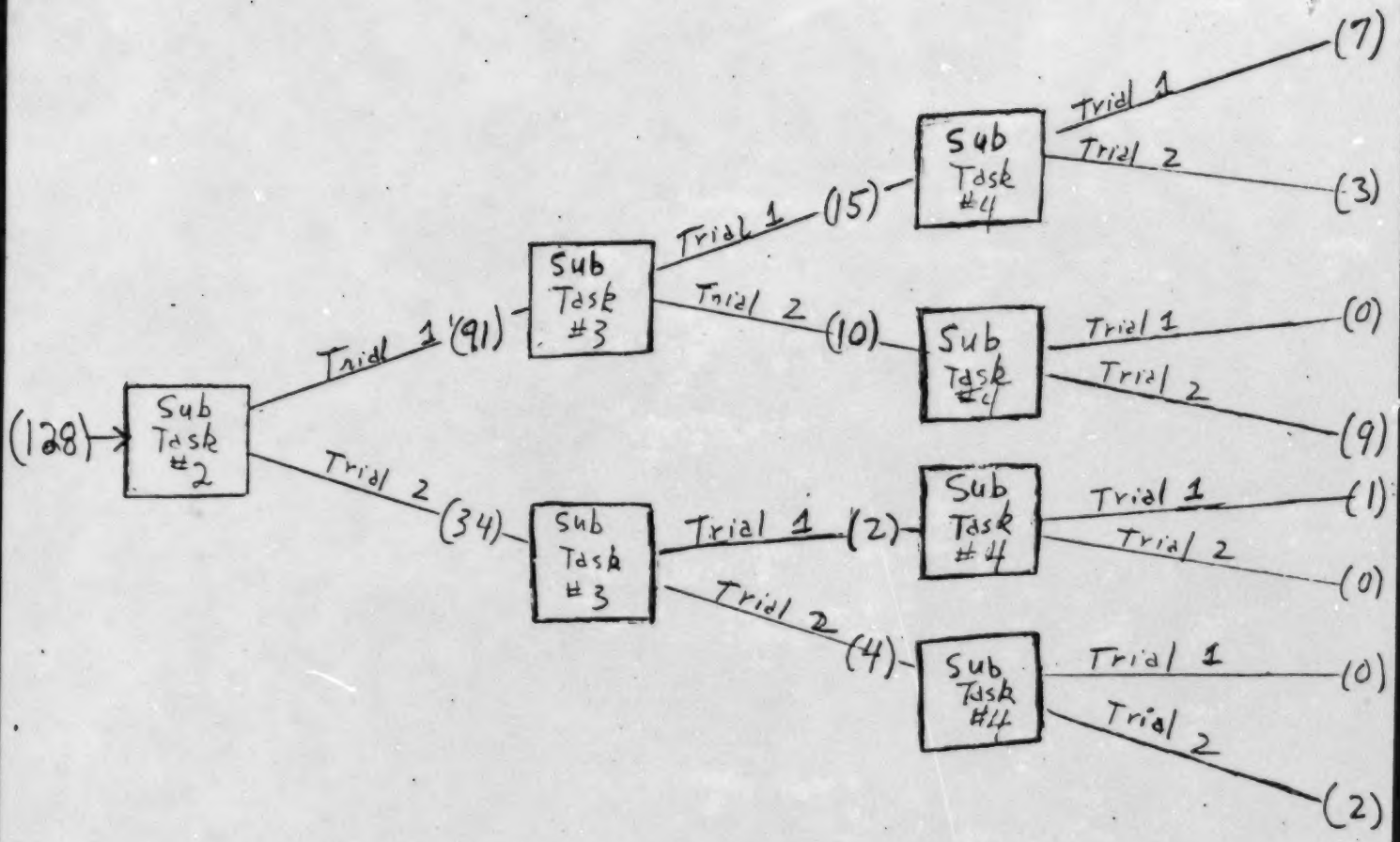

FIGURE 3: Number of Successes per Number of Trials on Subtasks #2 - #4 -- Study B.

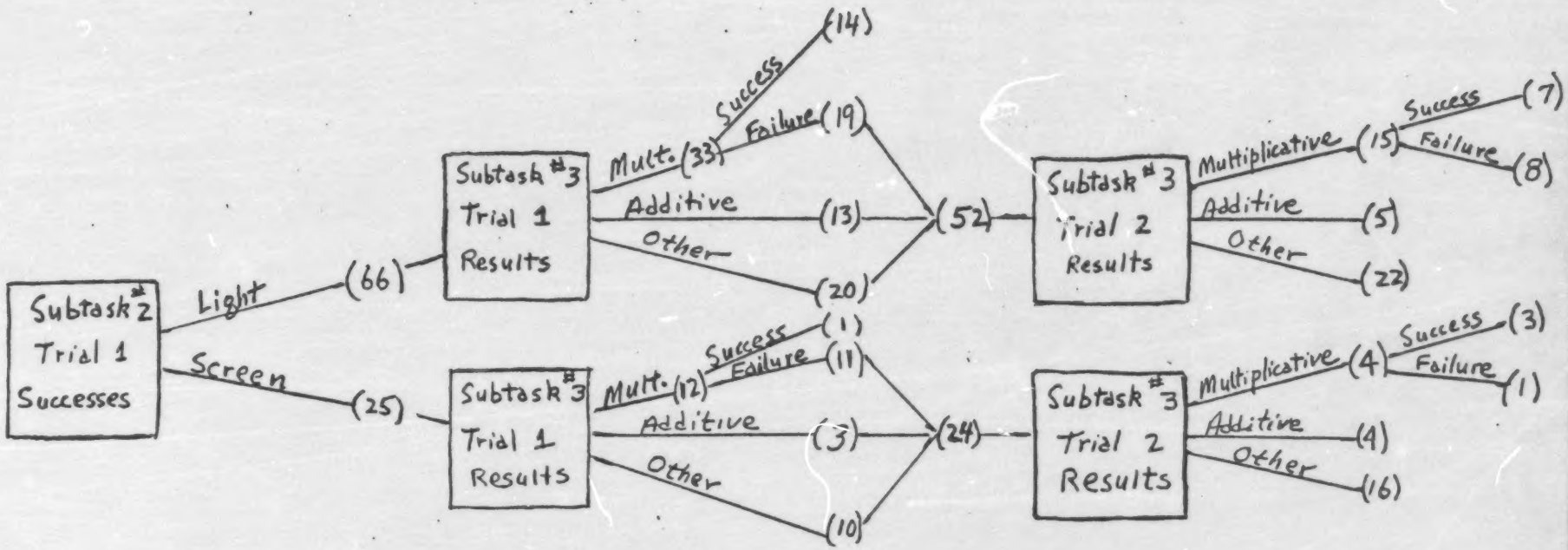

FIGURE 4: Pattern of Reasons in Subtask #3 for Light versus Screen as Reference Point -- Study B

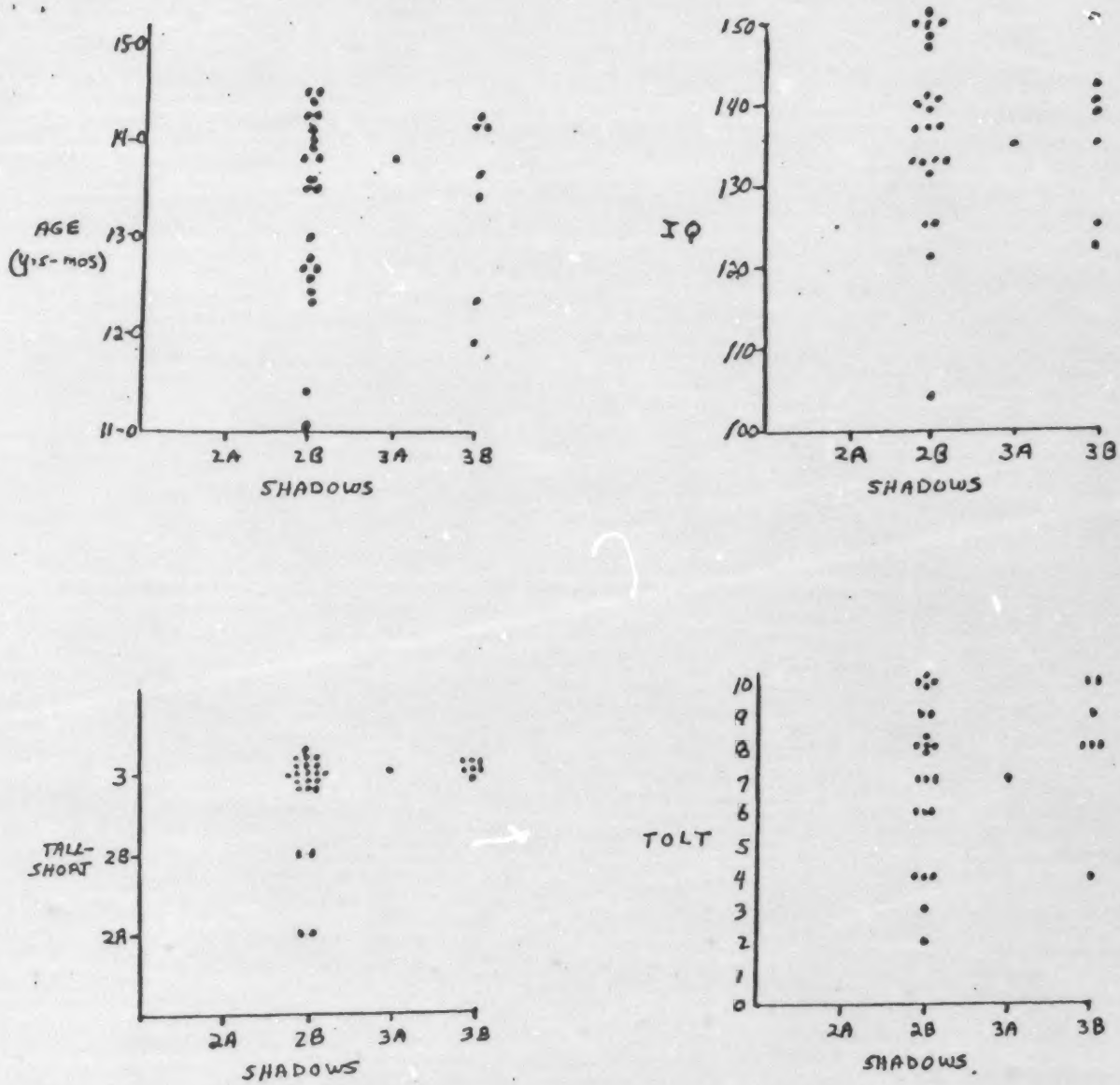

FIGURE 5: Scatter Plots of Shadows Results versus Other Variables -- Study C

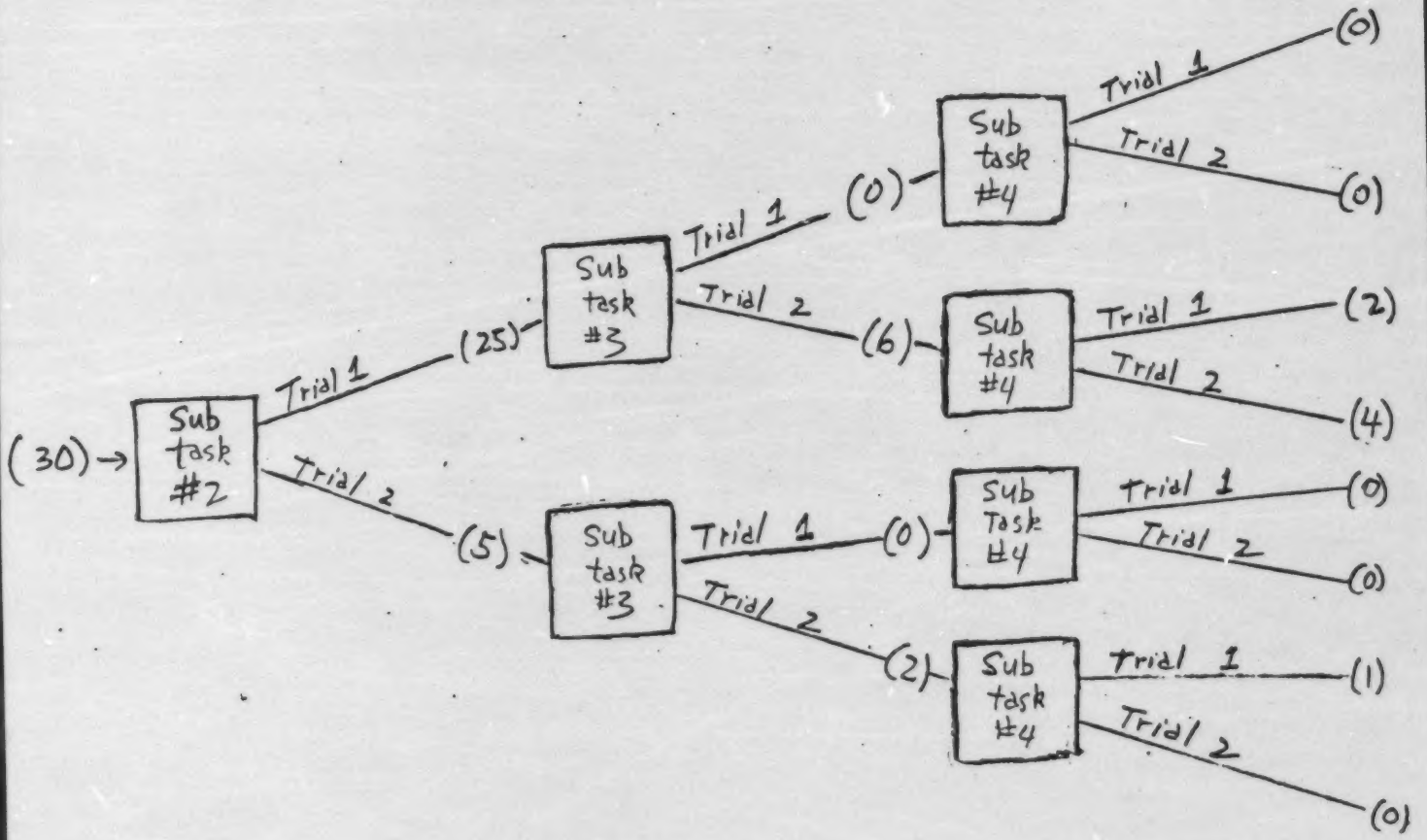

FIGURE 6: Number of Successes per Number of Trials on Subtasks #2 - #4 -- Study C

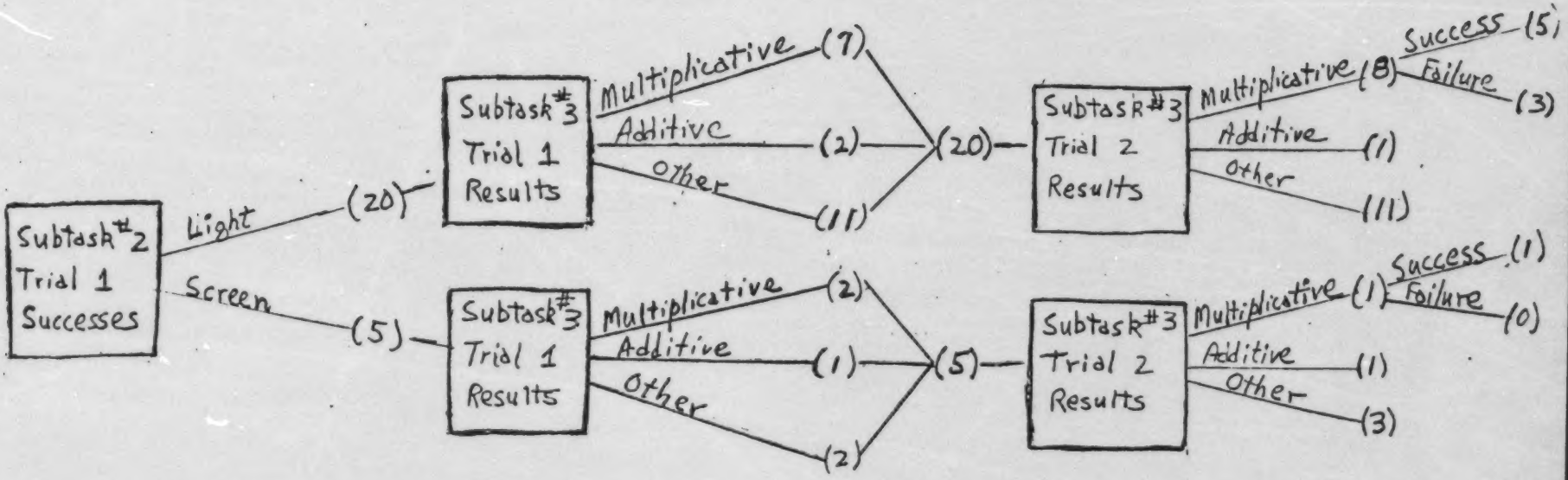

FIGURE 7: Pattern of Reasons in Subtask #3 for Light versus Screen as Reference Point -- C

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